Naval Research Laboratory

Washington, DC 20375-5320

Analysis of Intensity Structure of the ISI Model in the FAST2D Hydrocode

ANDREW J. SCHMITT

Laser Plasma Branch
Plasma Physics Division

August 28, 2001

20010925 262

REPORT DOCUMENTATION PAGE

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. **PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.**

1. REPORT DATE (DD-MM-YYYY)	2. REPORT TYPE	3. DATES COVERED (From - To)
August 28, 2001	Memorandum	Jan.-Sept. 2000

4. TITLE AND SUBTITLE	5a. CONTRACT NUMBER
Analysis of Intensity Structure of the ISI Model in the FAST2D Hydrocode	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)	5d. PROJECT NUMBER
Andrew J. Schmitt	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
Naval Research Laboratory
Washington, DC 20375-5320	NRL/MR/6730--01-8563

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR / MONITOR'S ACRONYM(S)
Department of Energy
1301 Clay Street
Oakland, CA 94612	DOE
	11. SPONSOR / MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release distribution is unlimited.

13. SUPPLEMENTARY NOTES

This work is referenced in the journal article "Growth of Pellet Imperfections and Laser Imprint in Direct ICF Targets," by A. Schmitt et al., Phys. Plasmas **8**, 2287 (2001).

14. ABSTRACT

The intensity fluctuation spectrum due to spatially and temporally incoherent laser light is examined in detail. Details of the implementation of an optically smoothed light model in numerical codes are discussed.

15. SUBJECT TERMS

Incoherent laser light, Numerical codes, Induced spatial incoherence, Spatial resolution

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE	UL	10	Andrew Schmitt
Unclassified	Unclassified	Unclassified			19b. TELEPHONE NUMBER (include area code) 202-767-3681

Analysis of intensity structure of the ISI model in the FAST2D hydrocode

Andrew J. Schmitt

Laser Plasma Branch, Plasma Physics Division

Naval Research Lab Wash. DC 20375

Consider a lens illuminated by an ISI beam and focused onto a target. Fourier optics tells us that the spatial distribution of the electric field at the lens is a direct analog of the wavenumber spectrum of the electric field at the target. For the moment, assume that the beam is normal to the target. The extent of the field in wavenumber space at the target is due entirely to the geometry of the focusing optics: the diameter of the lens D, the focal length of the lens f, or the F/# of the lens $F=f/D$. Assume further that the intensity profile of the beam at the lens is such that it uniformly covers the lens surface, is uniform in a statistical-time-average sense, and is characterized by the number of spatial coherence zones, N_z, that cover the lens surface. The resulting envelope of the electric field or intensity at the target will have a width $R_{spot} = N_z F \lambda_0$, or N_z times the diffraction limited spot size of the entire lens, $F\lambda_0$. The spatial width of the coherence zone at the lens surface is D/N_z. This electric field structure at the lens results in a spectral width of the electric field coherence zones at the target surface of $\Delta k_I \approx 2\pi/F\lambda_0$ as well as the total spectrum width there, $k_M = N_z \Delta k_I = 2\pi N_z/F\lambda_0$. Thus, the spectrum varies from $-k_M/2 < k < +k_M/2$ where k is the wavenumber of the field at the target.

(The preceding limits assume that the angle θ subtended by the lens is small enough that $\sin\theta \approx \theta$. More generally, the spatial coherence zone at position $-D/2 < x < D/2$ and width d at the lens will result in an electric field at the target at wavenumber $k=k_0\sin(\theta_x)$ (where $\theta_x=\text{atan}(x/f)$ and k_0 is the laser wavenumber $2\pi/\lambda_0$) and width $\Delta k_I = k_0(\sin[\text{atan}((x-d/2)/f)]-\sin[\text{atan}((x+d/2)/f)])$.)

Multiple Beams:

We can consider multiple beams in the limit that they are immediately adjacent to one another; this is equivalent to shortening the F/# of the incident lens optic to $F_{mult}=f/N_bD=F/N_b$ where F is the

LENS PLANE ELECTRIC FIELD (SPACE)
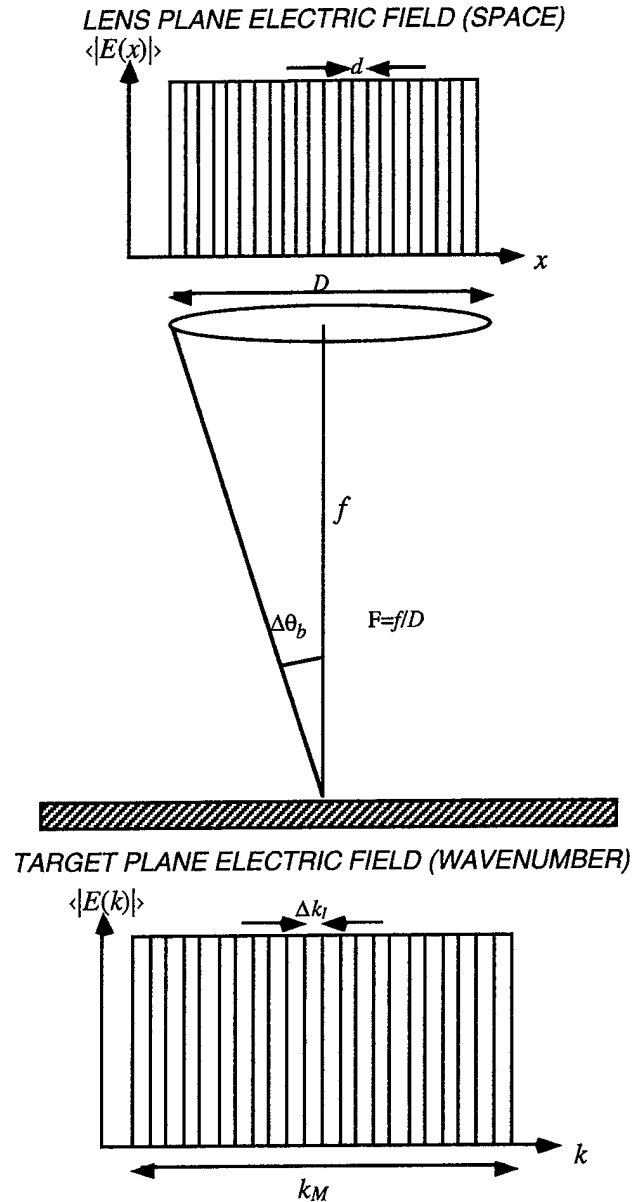
TARGET PLANE ELECTRIC FIELD (WAVENUMBER)

Fig. 1. Focusing optics produce the Fourier transform of the incident beam on the target.

F/# of the single lens. This results in the width of the k-spectrum of the electric field at the target increasing by the factor N_b, while the incoherent zone width Δk_I remains the same (the total number of coherence zones is multiplied by N_b). This leads to higher frequency structure in both the electric field and the intensity. When this shorter wavelength structure is averaged over a region larger than the smallest wavelengths, smoothing of the resulting beams occurs. We will consider this effect in more detail later.

I.1-1D: Intensity structure:

Given the k-spectrum of the electric field at the target, the intensity can be found as its convolution:

$$I(x) = E^*(x)E(x) = \int dk' e^{-ik'x} E^*(k') \int dk\, e^{ikx} E(k)$$

$$= \int dk' \int dk\, e^{ikx} E^*(k')E(k+k')$$

(1)

In the simulation code, the integral is effectively replaced by a summation (the Discrete Fourier Transform) as in:

$$I(k_\ell) = \sum_j E_j^* E_{j+\ell}$$

(2)

where $\ell = k_\ell/\Delta k_I$ varies from $-N_z+1$ to N_z-1, and the limits of the sum over j are from $j = 1-N_z/2-\text{MIN}(\ell,0)$ to $j = N_z/2-\text{MAX}(\ell,0)$.

Now if E_j are complex random values $r_1 + ir_2$ where r_1 & r_2 are instances of a Gaussian random distribution of width unity, then E_j is a vector in the complex plane with amplitude $\hat{a}$ and a uniformly distributed random phase between 0 and 2π. For any $\ell \neq 0$, $E_j^* E_{j+\ell}$ will also have a random phase and amplitude $\langle \hat{a} \rangle^2$. Summing n of these numbers will (on average) give a vector of amplitude $n^{1/2}\langle \hat{a} \rangle^2$ with a random phase. For each ℓ, the number of sums given by Eqn. (2) is simply $N_z-|\ell|$. Thus the (average) amplitude of the intensity from (2) can be written:

$$\langle I(k_\ell) \rangle = \left\langle \sum_j E_j^* E_{j+\ell} \right\rangle = \sqrt{N_z - |\ell|}\langle \hat{a} \rangle^2$$

(3)

where $\langle \rangle$ denotes an average element from the statistical ensemble. A special case exists for $\ell=0$ since the products $E_j^* E_j$ are all real and positive; the sum there is $N_z\langle \hat{a}^2 \rangle$. Finally, since the $\ell=0$ term also gives the average value of the intensity I_{avg}, we have: $I_{avg} = \int dx \langle I(x) \rangle / \int dx = \langle I(k=0) \rangle = N_z\langle \hat{a}^2 \rangle$; thus $\langle \hat{a}^2 \rangle = I_{avg}/N_z$. However, note the subtle distinction between $\langle \hat{a} \rangle^2$ and $\langle \hat{a}^2 \rangle$ -- they are not equal as the operations of ensemble-averaging and raising to an power *do not commute*. For the probability distribution function we consider here, $p(I) = \exp(-I/I_{avg})$, which gives $\langle I \rangle = I_{avg}$ and thus $\langle \hat{a}^2 \rangle = I_{avg}/N_z$ as noted above; but as $p(|E|) = 2(|E|/I_{avg})\exp(-|E|^2/I_{avg})$, $\langle |E| \rangle = \langle \hat{a} \rangle = \sqrt{(\pi\langle \hat{a}^2 \rangle)}/2 = (1/2)(\pi I_{avg}/N_z)^{1/2}$ (see Appendix A and fig. 2)

$$\langle I(k_\ell) \rangle = \sqrt{\frac{1-|\kappa_\ell|}{N_z}} \frac{\pi I_{avg}}{4}$$

(4)

where $\kappa \equiv k/k_M$. As we can see from fig. 2, this formula matches the simulation model very well, when averaged over a suitable time, and when the simulation is set up to resolve all the modes in the intensity spectrum.

Before leaving this section, I want to point out two things. First, note that the effect of adding multiple beams is to simply to increase the number of incoherence zones N_z by the factor N_b (number of beams), thus reducing each Fourier component of the intensity by the square root of the number of beams, as expected. However, multiple beams also multiply the extent of the intensity spectrum by N_b.

Secondly, note that the actual (time average) amplitude of an intensity perturbation at a given wavelength $\lambda = 2\pi/|k_\ell|$ is *twice* the quantity given in eqn. 4, because one must add both the positive and negative wavenumber contributions.

$|I(k)|$

Theory:
(eqn (4))
$N_Z=75$

Simulation, averaged over 1000 coherence times
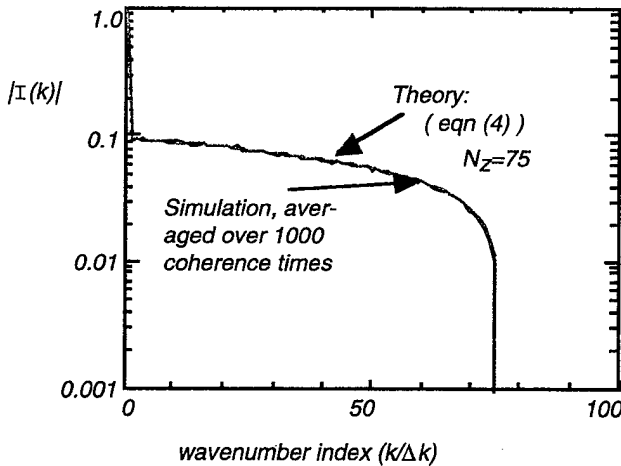
wavenumber index $(k/\Delta k)$

Fig. 2: The solid dark line shows the time averaged simulation results after 1000 coherence times, and the broken red line shows the analytic predictions, for $N_z=75$, $I_{avg}=1$ (eqn. 4). The wavenumber index corresponds to ℓ in the text.

I.2A-1D The problem of incomplete resolution: Inadequate Fourier resolution

It is a common problem in the simulation

that we cannot resolve all the modes generated by the laser beam. For instance, if a simulation has a transverse size (X_{len}) smaller than the spot size of the beam, the modes with wavelength $X_{len} < \lambda < R_{spot}$ (and others not fitting the periodicity of the mesh) cannot be resolved. In general, in a simulation of size X_{len} with N_x points along x (grid size $\Delta x = X_{len}/N_Y$) and periodic boundary conditions (needed by ISI and/or Fourier transforms, and in any case more general than predefined amplitude or gradient boundaries), the Fourier-space grid will have extent $-k_X/2 < k < k_X/2$ and resolution Δk_x where $\Delta k_x = 2\pi/X_{len}$ and $k_X = 2\pi/\Delta x = N_X \Delta k_x$. This resolution may not match the resolution needed for the field, e.g. in general, $\Delta k_x \neq \Delta k_I$. In particular we consider here the case of inadequate resolution such that $\Delta k_x > \Delta k_I$ (i.e., $X_{len} < R_{spot}$). Then each Fourier mode in the code must represent $N_r = \Delta k_x/\Delta k_I = R_{spot}/X_{len}$ modes (assume for the moment N_r is integer; the result will generalize to the non-integer case). A typical value of N_r in the code would be 7.5 ($R_{spot}=750\mu m$ [Nike], $X_{len}=100\mu m$).

There are many ways to model this. One is to simply (linearly) interpolate the real spectrum at the places where we *do* resolve the field (i.e., $k_{xi}=i\Delta k_x$, $i=-N_X,...,N_X$). Another way is to use the field averaged over the cell Δk_x in Fourier space.

Consider the first case, that of simple interpolation (i.e., the amplitude of the electric field is "sampled" at the point in k-space at which it is centered). Denoting $\acute{E}$ as the field resolvable in the simulation, we have: $\acute{E}(k_{xi})=E(k_I=k_{xi})$. The change in the number of modes, however, implies that the average intensity would change, unless we renormalize the field amplitudes, because we no longer have N_z modes but instead N_z/N_r modes. In this case $\langle\acute{E}\rangle^2 = \langle\hat{a}'\rangle^2 = (\pi I_{avg}/4)N_r/N_z$ and eqn (3) becomes:

$$\langle I(k_{xi})\rangle = \sqrt{N_z/N_r - |i|}\langle\hat{a}'\rangle^2 = \sqrt{\frac{1-|\kappa|}{N_z/N_r}}\frac{\pi I_{avg}}{4} \quad (5)$$

Consider the second way, and average the field amplitude over the Fourier grid cell Δk_x: $\acute{E}(k_{xi})=\int dk_x' E(k_x')/\Delta k_x = (\Delta k_I/\Delta k_x)\Sigma E(k_{xi}+n\Delta k_I)$, where the summation is over the N_r nearest modes. Assuming that the sum is again over statistically identical but mutually incoherent fields, $\langle\acute{E}(k_{xi})\rangle \sim N_r^{1/2}\langle E(k_x)\rangle = N_r^{1/2}\hat{a}$. $\hat{a}$ is still determined by the average intensity and the (total) number of

modes: $\hat{a}=(\pi I_{avg}/4N_z)^{1/2}$. However, we have also reduced the number of terms in the sum of eqn (2) to $N_z' = N_z/N_r$. Thus Eqn. (3) becomes:

$$\langle I(k_{xi})\rangle = \sqrt{N_z/N_r - |i|}N_r\langle\hat{a}^2\rangle = \sqrt{\frac{1-|\kappa|}{N_z/N_r}}\frac{\pi I_{avg}}{4}$$
$$(6)$$

...which is identical to eqn. (5). So in each case the necessity of normalizing the electric field amplitude to produce the correct average intensity, in tandem with our insistence that each electric field component is statistically identical, leads us to the same solution. Compared to the perfectly resolved case (eqn (4)), the individual intensity fluctuations are *larger by a factor of* $\sqrt{N_r}$. These formulas (5) or (6) represent what is in the FAST2D code now.

This increase in mean intensity fluctuation level is the result of preserving the total rms fluctuation and average intensity while keeping the ansatz that each electric field mode amplitude is identical. One way around this $\sqrt{N_r}$ increase in the amplitude would be to simply interpolate the mode amplitudes, but drop the constraint of producing the correct average intensity. We would find the intensity spectrum:

$$\langle I(k_{yi})\rangle = \sqrt{N_z/N_r - |i|}\langle\hat{a}\rangle^2 = \sqrt{\frac{1-|\kappa|}{N_zN_r}}\frac{\pi I_{avg}}{4} \quad (6a)$$

The correct average intensity can still be recovered by simply adding the appropriate (constant) amount to the DC mode ($k=0$): instead of $\langle I(k=0)\rangle = N_z\langle\hat{a}^2\rangle/N_r$, add the amount $I_{avg}(1-1/N_r)$. In this method, the individual intensity fluctuations are *smaller* by a factor of $\sqrt{N_r}$ compared to the resolved case (eqn (4)), *and a factor of N_r smaller than the current methods* (5) and (6).

Finally, the alteration in individual mode amplitude produced by the methods (5-6a) can be avoided by directly using the intensity spectrum given by eqn. (4) no matter what resolution we might have. Then (if needed) add the appropriate DC contribution to preserve the average intensity. This method preserves the average intensity and the individual intensity fluctuation level, at the expense of reducing the total rms intensity fluctuation level (as there are fewer modes represented).

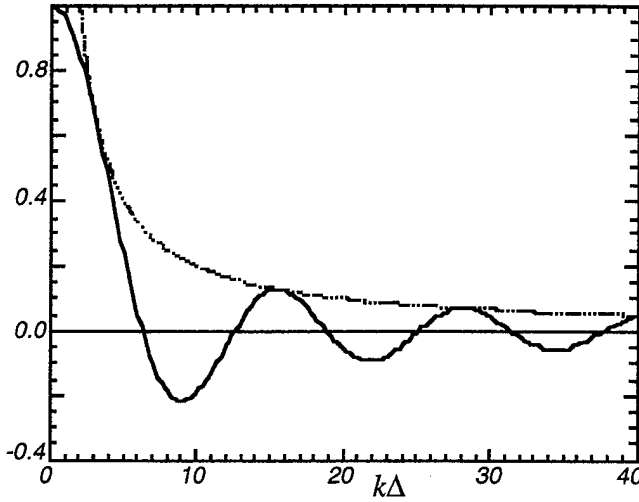(In the last two methods, there should probably be a random, statistical nature to this add-on

fig. 3: The solid line is the weighting factor $2\sin(k\Delta/2)/k\Delta$ as a function of $k\Delta$, and the dotted line is its envelope, $2/k\Delta$.

constant -- since this DC term is actually being made up of all from those N_r-1 modes that we failed to resolve during the interpolation. Also, it may be appropriate to reduce the mean variance of the individual mode amplitude (by ~N_r-1/2) to reflect the effect of averaging the adjacent unresolved modes.)

I.2B-1D:The problem of incomplete resolution: Inadequate spatial resolution

Another common problem, particularly when using multiple beams, is not having enough spatial resolution to resolve the high frequency spatial structure (remember, the smallest intensity wavelength is expected to be of order $F\lambda_0$, where F in this case is represents the total beam cluster). Let us denote Δ as our smallest resolvable spatial extent (n.b.: Δ may be the same size as --or larger than-- the mesh spacing Δx). Then the resolved intensity at any scale due to this minimum resolution is:

$$I_R(x) = \frac{1}{\Delta}\int_{x-\Delta/2}^{x+\Delta/2}dx'I_E(x) = \sum_k I_k \int_{x-\Delta/2}^{x+\Delta/2}dx'e^{ikx'}$$

$$= \sum_k I_k e^{ikx}\left\{\frac{2\sin k\Delta/2}{k\Delta}\right\} \qquad (7)$$

where the subscript "R" refers to the resolved intensity and the subscript "E" refers to the exact intensity, and I_k are the Fourier coefficients of the "exact" intensity. Thus, averaging the exact intensity over the distance Δ is equivalent to multiplying

the Fourier coefficients of the intensity by the factor $2\sin(k\Delta/2)/k\Delta$. (see fig. 3). Note that at $k\Delta=n\pi$ for integer n greater than zero, the multiplier is zero.

What about the high-frequency end of the k spectrum that can't be resolved in the simulation, e.g. $|k|>\pi/X_{len}$? These values of k are *aliased* to values of k that *do* appear in the simulation. The rule for such aliasing is that the actual k is mapped to a resolvable k given by:

$$k_{RES} = \tfrac{1}{2}k_M - \left|(k\text{ MOD }k_M)-\tfrac{1}{2}k_M\right| \qquad (8)$$

where MOD is the modulo function. Thus the intensity at each resolvable wavenumber, k, is made up of the weighted sum of the intensity at wavenumbers k, k_M-k, k_M+k, $2k_M$-k, ...

II.A-2D: The actual (two dimensional) illumination spectrum

Now we concern ourselves with the actual illumination spectrum with two transverse dimensions (i.e., the "real world" case). Again, we assume that the k-spectrum of the electric field at the target is known (as it is simply equivalent to the spatial-distribution of the electric field at focusing lens). And again, as in the development of eqn. (1), the intensity can be found as its convolution:

$$I(x,y) = E^*(x,y)E(x,y)$$

$$= \iint dk_x'dk_y'e^{-ik_x'x-ik_y'y}E^*(k_x',k_y')\times$$

$$\iint dk_xdk_ye^{ik_xx+ik_yy}E(k_x,k_y) \qquad (9)$$

$$= \iint dk_x'dk_y'\iint dk_xdk_ye^{ik_xx+ik_yy}\times$$

$$E^*(k_x',k_y')E(k_x+k_x',k_y+k_y')$$

and now the discrete summation is also in two transverse directions:

$$I(k_\ell,k_n) = \sum_m\sum_j E^*_{j,m}E_{j+\ell,m+n} \qquad (10)$$

This summation is determined by the number of modes in the sum, since each summand is statistically identical. The number of sums is given (geometrically) by the area of overlap of two circles of width $k_M/2$, separated by a distance $|k|=\sqrt{(k_x^2+k_y^2)}$. This area of overlap is the number of sums in eqn. (10) (in units of Δk^2 (we assume $\Delta k \equiv \Delta k_x = \Delta k_y$)), and is given by:

$$N_s = \frac{4}{\Delta k^2} \int\limits_{k/2}^{k_M/2} dx \sqrt{k_M^2/4 - x^2} \qquad (11)$$

or:

$$N_s = \frac{1}{4}\left(\frac{k_M}{\Delta k}\right)^2 \left\{\pi - 2\sin^{-1}[\kappa] - 2\kappa\sqrt{1-\kappa^2}\right\} \qquad (12)$$

where $\kappa \equiv k/k_M$ varies from 0 to 1. So if each individual electric field amplitude is â, the amplitude of the intensity fluctuation at a given $\kappa = \sqrt{(k_x^2+k_y^2)}/k_M$ is:

$$\left\langle I(k_\ell, k_n) \right\rangle = \left\langle \sum_m \sum_j E_{j,m}^* E_{j+\ell,m+n} \right\rangle = \sqrt{N_s(\kappa)}\langle \hat{a}\rangle^2 \qquad (13)$$

Again, the special case exists at $\kappa=0$, where the sum is coherent and the mean intensity is the average intensity: $N_s(\kappa=0)\langle\hat{a}^2\rangle = (k_M/\Delta k)^2\pi\langle\hat{a}^2\rangle/4 = I_{avg}$. This determines $\langle\hat{a}^2\rangle$: $\langle\hat{a}^2\rangle = 4I_{avg}/\pi N_z^2$, $(N_z=k_M/\Delta k)$ and by extension (see discussion previous to eqn. (4) and Appendix A) $\langle\hat{a}\rangle^2 = I_{avg}/N_z^2$. So the average intensity fluctuation at wavelength κ is given by:

$$\langle I(\kappa)\rangle = \sqrt{\pi - 2\sin^{-1}[\kappa] - 2\kappa\sqrt{1-\kappa^2}} \; \frac{I_{avg}}{2N_z} \qquad (14)$$

where $N_z=k_M/\Delta k$ is the number of coherence zones measured across a 1D slice of the lens (the total number of coherence zones is $\sim N_z^2$). Note that for small (but finite) κ, $\langle I(\kappa\Rightarrow 0)\rangle \Rightarrow \sqrt{\pi}/(2N_z)$. This

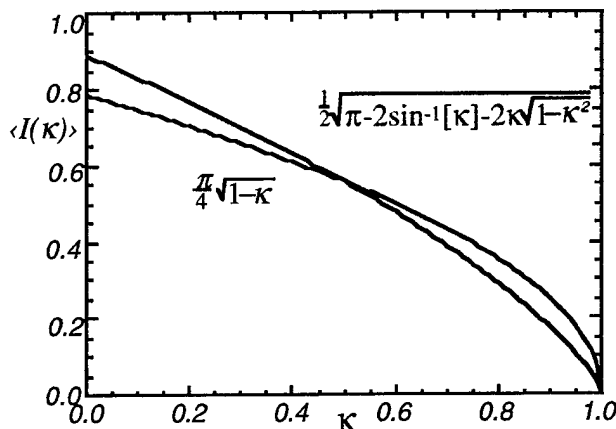
fig. 4: The 2D spectrum of intensity fluctuations as a function of wavenumber $\kappa=sqrt(k_x^2+k_y^2)/k_M$ (in red). The 1D spectrum (eqn.4) is shown for comparison; both use $N_z=1$ for comparison purposes.

spectrum as a function of κ is very similar to the 1D case, except that it's smaller (per mode) by a factor $\sim N_z^{1/2}$.

II.B-2D: Properly Accounting for 2D effects with a 1D treatment

The major difference between the 2D and the 1D treatment of ISI is that the number of modes with large $|k|$ is larger in 2D: the number of modes with a given wavenumber $|k|$ increases linearly as $\sim |k|$. One (arbitrary) way to take this into account in 1D is to preserve the relative σ_{rms} (compared to the full 2D distribution) at a given wavenumber. The standard deviation of the 2D distribution is calculated straightforwardly (using eqn. (14), and assuming a round focusing lens):

$$\langle \delta I \rangle^2 = \sum_{k_x\neq 0}\sum_{k_y\neq 0}\langle I(k_x,k_y)\rangle^2 \approx \int\limits_{\Delta k}^{k_M} dk\, 2\pi k \langle I(k)\rangle^2 \Big/ \Delta k^2$$

$$= \frac{\pi}{2} I_{avg}^2 \int\limits_{1/N_z}^{1} d\kappa\, \kappa\left(\pi - 2\sin^{-1}[\kappa] - 2\kappa\sqrt{1-\kappa^2}\right)$$

$$= \frac{\pi}{2} I_{avg}^2 \left[\begin{array}{l} \frac{\pi}{2}\kappa^2 - \left(\kappa^2 - \frac{1}{4}\right)\sin^{-1}[\kappa] \\ \quad - \frac{\kappa}{2}\left(\kappa^2 + \frac{1}{2}\right)\sqrt{1-\kappa^2} \end{array}\right]_{1/N_z}^{1}$$

$$\cong \frac{\pi^2}{16} I_{avg}^2 \qquad (15)$$

The integrand is thus simply proportional to the σ_{rms} for each mode in the 2D distribution. For comparison, the 1D distribution gives (using eqn. 4):

$$\langle \delta I \rangle^2 = \sum_{k\neq 0}\langle I(k)\rangle^2$$

$$\approx \left(\int\limits_{-k_M}^{-\Delta k} dk\langle I(k)\rangle^2 + \int\limits_{\Delta k}^{k_M} dk\langle I(k)\rangle^2\right) \Big/ \Delta k$$

$$= 2\int\limits_{\Delta k}^{k_M} dk\langle I(k)\rangle^2 \Big/ \Delta k \qquad (16).$$

$$= \frac{\pi^2}{8} I_{avg}^2 \int\limits_{1/N_z}^{1} d\kappa\,(1-\kappa) = \frac{\pi^2}{16} I_{avg}^2\left(1 - \frac{2}{N_z} + \frac{1}{N_z^2}\right)$$
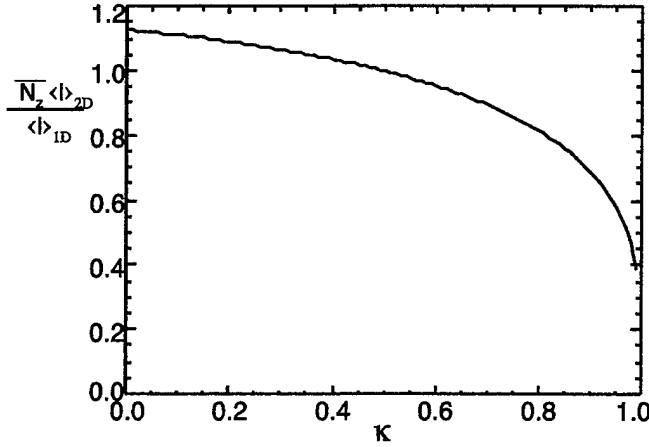
$$\cong \frac{\pi^2}{16} I_{avg}^2$$

fig. 5: The function converting 1D imprint to 2D, as a function of wavenumber κ (c.f. eqn. 17).

The total σ_{rms} in 2D is the same as in 1D, but since there are many more modes in 2D, the typical σ_{rms} in any given individual mode is much lower.

The 1D distribution can be "renormalized" to 2D by requiring that (*i.a*) the individual 1D mean-intensity fluctuation in each mode is equal to the 2D mean-intensity-fluctuation; and (*i.b*) the 1D total σ_{rms} is the same as the 2D σ_{rms}. Alternatively, we can (*ii*) create a new 1D distribution by averaging the 2D distribution in k-space to obtain a "representative" distribution.

Method (*i*) would be used to scale a 1D (transverse) simulation to a 2D (transverse) simulation; in this case the imprint at each wavenumber of a 1D (transverse) simulation would be scaled by the multiplier composed of eqn(14)/eqn(4):

$$\langle I(\kappa)\rangle_{2D} = \frac{2}{\pi}\sqrt{\frac{\pi - 2\sin^{-1}[\kappa] - 2\kappa\sqrt{1-\kappa^2}}{N_z(1-\kappa_x)}}\langle I(\kappa_x)\rangle_{1D}$$

(17)

(see figure 5); this multiplier is of order unity except for the $N_z^{-1/2}$ factor. Using this expression to renormalize the mode amplitudes and summing them over the resulting 2D distribution also yields the correct total σ_{rms}.

Method (ii), creating a new 1D distribution by averaging the 2D distribution in |k|-space, could be used if we wished to generate a 2-D like modal σ_{rms} in a 1D (transverse) simulation directly. Requiring that the intensity fluctuation amplitude at κ_x reflect the total 2D fluctuation level at κ, the 1D "equivalent" of the 2D field would be:

$$\langle I(\kappa_x)\rangle =$$

$$\sqrt{\kappa_x\left(\pi - 2\sin^{-1}\kappa_x - 2\kappa_x\sqrt{1-\kappa_x^2}\right)}\sqrt{\frac{\pi}{4N_z}}I_{avg}$$

(18)

This has been normalized by requiring that the total σ_{rms} be the same as in 2D. It's not clear how to get this spectral shape simply by generating the electric fields as we did before -- the electric field components are no longer independent of κ_x. Probably the easiest way is to assemble the field with the typical 1D intensity distribution (sec. I), then apply a 1D-to-2D transform in Fourier space by using the ratio of eqn (18) to eqn (4) (see also fig. 6):

$$\frac{\langle I(\kappa_x)\rangle_{2D-mock}}{\langle I(\kappa)\rangle_{1D-sim}} = \sqrt{\frac{4\kappa_x\left(\pi - 2\sin^{-1}\kappa_x - 2\kappa_x\sqrt{1-\kappa_x^2}\right)}{\pi(1-\kappa)}}$$

(19)

(which is just $\sqrt{\pi N_z \kappa_x}$ times eqn. (17)). As before, missing modes (due to incomplete resolution in Fourier space) can be accounted for by normalizing the resulting field to ensure that the total σ_{rms} is
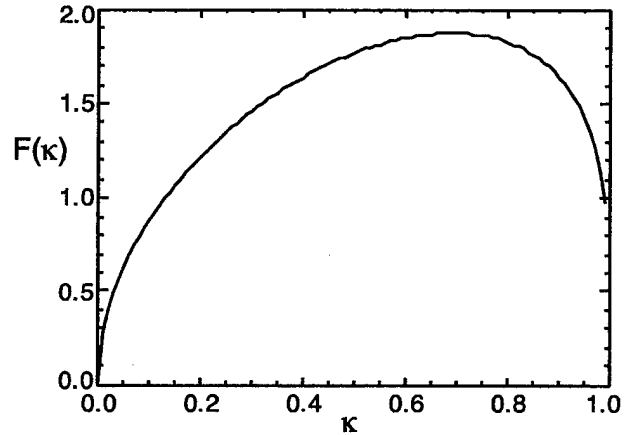

fig. 6: The normalization function (eqn (19)) applied to a 1D distribution spectrum, in order to get a 2D intensity distribution spectrum.

preserved.

Acknowledgement

08 September 2000

Appendix A

The distribution function of the electric field magnitude can be derived from the distribution function of the intensity using the cumulative distribution function (e.g., B.W. Lindgren *Statistical Theory* (2nd Ed.) p. 69 (1968), or any basic text on statistics). In general, we can equate the two cumulative distribution functions (e.g., $F(X)$), which are defined from the (density) distribution functions (e.g., $p(x)$) as:

$$F(X) \equiv \int_{-\infty}^{X} dx\, p(x) \qquad (A.1)$$

In the case under consideration here, we know the intensity distribution function $p(I)$, and we wish to find the electric field distribution function $p(|E|)$. Since $p(I) = \exp[-I/I_{avg}]/I_{avg}$, eqn. (A.1) gives $F_I(I) = 1 - \exp[-I/I_{avg}]/I_{avg}$, which is equal to the cumulative electric-field distribution: $F_{|E|}(|E|) = 1 -$ $\exp[-|E|^2/I_{avg}]/I_{avg}$. $(I_{avg} \equiv \langle I \rangle.)$ The probability density function $p(|E|)$ is found from the cumulative distribution through differentiation as:

$$p_{|E|}(|E|) = \frac{dF_{|E|}}{d|E|} = 2\frac{|E|}{\sqrt{I_{avg}}} e^{-|E|^2/I_{avg}} \qquad (A.2)$$

(This has been normalized using the requirement that the total probability is unity). Furthermore, the average value of $|E|$, $\langle |E| \rangle$ is found as the first moment of the probability density function:

$$\langle |E| \rangle = \int_0^{\infty} d|E|\,|E|\, p(|E|) = \frac{\sqrt{\pi}}{2}\sqrt{I_{avg}} \qquad (A.3)$$

We note that, although the average value of the intensity distribution is (by definition) I_{avg}, the average value of the electric field distribution is *not* simply $I_{avg}^{1/2}$, but is instead only about 89% of that value.

Attachment A

The IDL function implementing the ISI model, similar to that used in the FAST2D code, and used for the comparisons above (fig. 2).

```
function ISIS2, Nbeams, Nmodes, NMX, seed, AVINT=AVINT, debug=debug, $
        nt1=nt1, verbose=verbose, eef=eef
;+
; NAME:
;   isis2
;
; CALLING SEQUENCE:
;   intensity = isis2(Nbeams, Nmodes, Nmax, SEED)
; PURPOSE:
;   generate an ISI profile of length NMX, return in array IFlux
;
; FUNCTION RETURNS:
;   IFLUX:      array of size NMX that is output as isi intensity
;
; PARAMETERS:
;   Nbeams:    Number of beams that we'll be adding (?)
;   NMODES:    Integer number of ISI modes (in k space) of EACH beam
;              curiously, they will be arranged from k=[0 : NMODES-1],
;              e.g. biased to k.ge.0 in Fourier space. Not that this should matter.
;   NMX:       Integer size of output Iflux vector
;   seed:      Random number seed. (should be provided so that it isn't reset to same
;                value on every call)
;
; OPTIONAL PARAMETERS
;   Avint:     Average intensity (Def = 1.0)
;   nt1:       Number of total modes per resolved mode (def = 1.0)
;                (In practice, this will be Y_focal/Y_length)
;   verbose:   Verbose flag (def=0); if nonzero, debugging notices get printed
;   debug:     debug flag (def = 0)
;   eef:       The electric field
;
; HISTORY:
;   programmed circa 04 Nov 1999 : in preparation for APS/DPP 99 confernce
;   modified from isis.pro on 04 Jan 2000 : testing out new ideas of
;   multiple beams & averaging.
;-
if n_elements(avint)   eq 0 then avint   = 1.0
if n_elements(debug)   eq 0 then debug   = 0
if n_elements(nt1)     eq 0 then nt1     = 1.0
if n_elements(verbose) eq 0 then verbose = 0
; --------------------------
```

```
NmodeTot = Nmodes * Nbeams
; find Np = 2**MM such that Np >(2*NmodeTot) .and. Np >= Nmx
; then Nspace = Np/Nmx
MM = FIX( ALOG(NmodeTot)/ALOG(2.) + 2 ) > FIX( ALOG(Nmx)/ALOG(2.))
if (NP lt Nmx) then NP = NP*2
NSPACE = NP/NMX   ; I think this is guarranteed to be integer
if verbose ne 0 then begin
    print,'In isis2: using vector of length ',np,' to generate length ',nmx
    print,' -- nspace = ',nspace,' averaging points'
endif
; --------------------------------------------------------------------
; construct a random amplitude distribution in Fourier space
; for wavenumbers corresponding to indices NP/4 to NP/4+NmodeTot
; --------------------------------------------------------------------
eef = complexarr(np)
eef(0) = CoMPLeX(randomn(seed, Nmodetot), randomn(seed, Nmodetot))
; add in effects of other modes not resolved here
if (nt1 gt 1.0) then begin
    for ijk = 2, nt1 do begin
        eef(0) = eef(0:*) + CoMPLeX(randomn(seed, Nmodetot), randomn(seed, Nmodetot))
    endfor
    if nt1 mod 1.0 gt 1.E-4 then begin
        eef(0) = eef(0:*) + $
            (nt1-fix(nt1))*CoMPLeX(randomn(seed, Nmodetot), randomn(seed, Nmodetot))
    endif
endif
eef = eef/sqrt(2.*Nmodetot*nt1)
; --------------------------------------------------------------------
;  NOTE: Normalization of 1./NP is applied in direction=-1
; (from IDL online help:
;        A normalization factor of 1/N, where N is the
;        number of points, is applied during the forward
;        transform.)
sav = abs( NP*fft(eef, -1) )^2
; --------------------------------------------------------------------
;  average every Nspace points, put it into IFlux
indx = indgen(Nmx)*nspace
Iflux = sav(indx) ;  > 0.
if Nspace gt 1 then begin
  for ijk = 1, Nspace-1 do begin
     Iflux = Iflux + sav(indx + ijk)
  endfor
  Iflux = Iflux / float(Nspace)
endif
; --------------------------------------------------------------------
Iflux = Iflux * avint
; --------------------------------------------------------------------
if debug ne 0 then stop
return, Iflux
END
```